# Multigrid Preconditioning in Fully-Implicit Evolution of the Ocean

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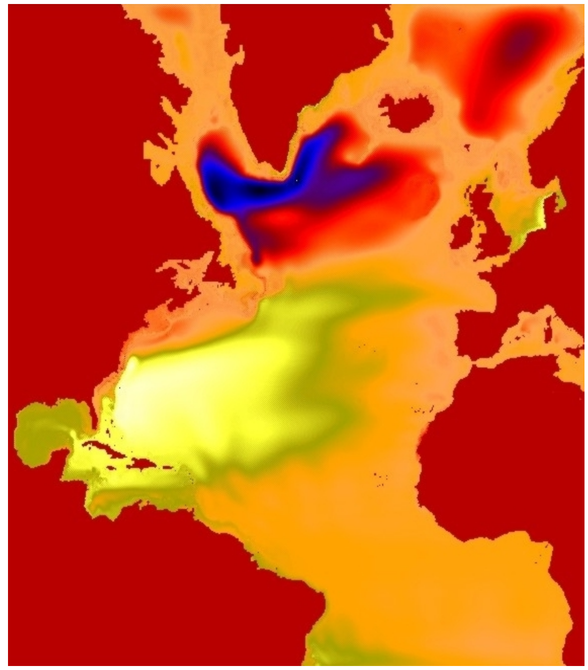
## Introduction

The ocean plays a significant role in the earth's climate. For example, the North Atlantic is one of the main sources of deep water, and through thermohaline circulation (global density-driven flow; temperature and salinity determine the density of water), is strongly coupled to the global climate. Some models predict that a weakening of thermohaline circulation in the North Atlantic, owing to doubled $CO_2$ levels in the atmosphere, will ultimately shift our climate to ice-age conditions. However, quantifying the uncertainty in predicted variations of mesoscale ocean circulation requires accurate quantitative simulations of complex multiscale processes. Specifically, thermohaline processes adjust on the centennial-millenial time-scale. Mid-latitude planetary waves equilibrate on a time scale of years, and even faster barotropic adjustment takes place on the order of days. Moreover, within this hierarchy, modes at each scale are strongly influenced by the dynamics of faster modes. Thus, to address the need for efficient long-time studies of this multiscale system we are developing optimal multilevel iterative solvers for fully-implicit time stepping algorithms.

## Implementation and Solvers

The accuracy and stability properties of the discrete time stepping algorithm are limiting factors in large-scale simulations. Explicit time advancement is naturally limited by the need to resolve the fastest dynamics, namely the barotropic modes. Time splittings help somewhat, as these handle the barotropic component implicitly, and the baroclinic component explicitly. Yet, even in this case the time-step restriction prohibits effective studies of centennial-millenial dynamics at a sufficiently high spatial resolution. In contrast, a fully implicit approach can relax this time step restriction. The time step can be chosen to resolve the scales of interest, namely the centennial-millenial dynamics of thermohaline circulation. The faster scales are not ignored or eliminated, instead they are continually adjusted to the correct equilibrium which in turn forces an equilibrium solution for the slow modes.



*The sea surface height in the North Atlantic is color coded such that white is highest and black is lowest. The whitish region to the East of the south-eastern seaboard of the US is the subtropical gyre and its western boundary marks the Gulf Stream. The wind-driven circulation is simulated here on a 0.2 degree grid using a shallow layer representing the upper ocean overlying a deep quiescent layer representing the ocean below the thermocline, and with a fully implicit time stepping scheme. Reanalysed, time-averaged, steady winds are used.*

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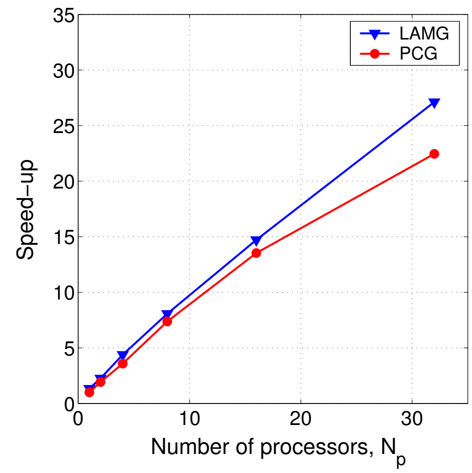
Implicit time advancement is currently implemented in a branch of the Parallel Ocean Program (POP), dubbed POPJr. The implementation is based on a Jacobian-Free Newton-Krylov (JFNK) algorithm. Due to the strongly-coupled, multi-scale dynamics of the physical problem, the discretized equations are stiff and preconditioning of the Krylov iterations is necessary. Physics based preconditioners, which are based on an approximate Schur complement of the semi-discretized (in time) model, are a matter of current research. One such preconditioner is the semi-implicit discretization, in which an elliptic equation governs the sea surface height. The original implementation of POPJr used Preconditioned Conjugate Gradient (PCG) to solve this elliptic equation, and consequently, both semi- and fully-implicit algorithms scale poorly with increasing spatial resolution. Furthermore, PCG is only applicable to symmetric positive definite systems. For general nonsymmetric preconditioners, such as those that result from the implicit treatment of the Coriolis forcing, a more flexible, and scalable alternative is desirable.

To address these issues we used the adaptive hierarchical solution method implemented in the Los Alamos Algebraic Multigrid (LAMG) package. This flexible multilevel algebraic algorithm provides a scalable linear solver that has been tested on various large-scale parallel machines, including the 4096 processor ASC-Q [1]. In this work, LAMG was successfully employed in both the semi- and fully-implicit formulations of POPJr. A comparison of convergence and scalability of LAMG versus PCG is shown in the figure below. The excellent convergence rate obtained in this setting in conjunction with the scalability of LAMG will allow for considerably finer meshes to be considered in the future.

## Conclusions

The implicit algorithm, with LAMG, will allow for high resolution, scalable, three-dimensional simulations. In the simulation presented here we focused on wind-driven circulation in the North Atlantic, the distinguishing feature of which is the gulf stream. The separation of the gulf-stream from the east coast of the US is a long-standing issue of great interest and we are now in a position to examine the dependence of this separation on the Reynold's number.




*Speed-up is defined as the time per Newton iteration using $N_p$ processors, divided by the time for a single processor using PCG. LAMG is slightly faster on a single processor and, even for this small problem, scales better than PCG.*

## References


[1] W. JOUBERT AND J. CULLUM. Scalable algebraic multigrid on 3500 processors. Technical Report LAUR-03-568, Los Alamos National Laboratory, Los Alamos, NM, 2003.

[2] B. NADIGA AND M. TAYLOR. Fully implicit time stepping for ocean modeling. pre-print.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